Relaxation time hierarchy in a two-component quasiparticle gas

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A quasiparticle description of various condensed media is a very popular tool in the study of their transport and thermodynamic properties. I present here a microscopic theory for the description of diffusion processes in a two-component gas of quasiparticles with arbitrary dispersion law and statistics. In particular, I analyze the role of interaction within each subsystem (i.e., between identical quasiparticles) in relaxation of the whole system. The approach for solving such kinetic problems allows one to study the most important limiting cases and to clarify their physical sense. Classical results for diffusion coefficients of light particles in a massive gas (Lorentz model) and of massive particles in a light gas (Rayleigh model) are obtained directly from the general solution without using artificial approaches, as was done earlier. This provides a possibility to generalize these popular models on quasiparticle systems.

I. INTRODUCTION

It is well known that a variety of properties of some condensed media can be described by interaction processes in quasiparticle gases. These are, say, transverse and longitudinal phonons in solids, phonons and magnons in magnetically ordered materials, phonons and rotons in superfluid helium, conduction electrons and holes in semiconductors, etc. As a rule, to study the dissipative properties of such systems investigators use either the classical kinetic theory in its simplest form or some semi-intuitive models, which lead sometimes to quite ambiguous results in the case of quasiparticle systems. Here I present a theory of diffusion processes in two-component quasiparticle systems, which, in general, is independent of particular quasiparticles statistics and dispersion law. Let me first briefly review the present state of the classical kinetic theory and analyze its limitations in the description of quantum quasiparticle systems of condensed media.

The kinetic theory of gases in modern understanding is attributed to the pioneering work by Maxwell [1], in which he has proved the law for distribution of velocities of molecules in a homogeneous equilibrium gas (the so-called Maxwell velocity distribution) and the law of equidistribution of average energy of molecules in a mixture of gases. His results were updated and improved in further works devoted to the theory of inhomogeneous gases (for a history of the problem see Ref. [2]). However, as a basis of all mathematical methods of the modern kinetic theory it is necessary to consider the basic works by Boltzmann [3], in which the H theorem was proved and the classical Boltzmann equation was introduced.

The Boltzmann equation is an integro-differential equation describing the collisional behavior of a rarefied gas. Until now it remains a basis of the kinetic theory of gases and appears to be very fruitful not only for a research of classical gases, which Boltzmann himself kept in mind, but—with an appropriate generalization—for the study of electron transport in solids and plasma, transport of neutrons in nuclear reactors, phonon and roton transport in superfluid liquids, and transport of a radiation in atmospheres of stars and plan-

ets. For the past 130 years these researches have led to significant achievements both in the new areas, and in old ones [4].

Generally, a kinetic equation of the Boltzmann type (the equation describing the evolution of a single-particle distribution function in an phase space) represents the integro-differential equation, where remarkable property is the non-linearity of the collision term. Just this fact makes an obstacle in the construction of methods for solving the kinetic equation. The monographies [2,5] are devoted to a detailed exposition of such methods in the case of classical gas systems.

In the majority of experimental problems there is no necessity to use the detailed microscopic description of gas systems at the level of distribution functions. As a rule, investigation of physical processes in macroscopic systems is carried out at the less detailed level of hydrodynamic variables. Since these variables are determined through the moments of a distribution function, then, as a rule, a detailed study of the main moments of the distribution function appropriate to collisions invariants is required, but not the distribution function itself. Thus, the connection between the kinetic theory and hydrodynamics appears to be one of the main problems. In particular, one of the main aspects of this problem is the determination of transport coefficients, such as the diffusion coefficient and viscosity (first and second), thermal conductivity appeared in equations of hydrodynamics of a viscous liquid [6].

In spite of the long history of physical kinetics, today we have a rather small number of approaches to a solution of the kinetic equations. All these approaches and methods these were formed depending on concrete problems, on which were directed. Most general methods of research of nonequilibrium state of classical and quantum gases were directed in demonstrating a mathematical resolvability (or insolubility) of certain basic problems in principle, rather than on construction of serviceable theories, suitable for deriving a solution of concrete physical problems.

The classical methods of deriving a solution of the kinetic equations allow one to derive the kinetic coefficients as series expansions on an infinite set of orthogonal polynomials.

However, it appeared to be very difficult to use these classical results for numerical calculation and analysis of physical processes in real systems. This is caused by impossibility of selection of the contribution from different types of interactions to various kinetic coefficients, while many physical systems behave qualitatively differently for different ratio between the speed of a relaxation inside each subsystem and between subsystems.

In classical gases with pointlike interaction, this problem is less important because the speed of relaxation inside subsystems is unambiguously determined by their mass and concentration. In the quantum case, when we talk about quasiparticles, the situation becomes more complicated. The mechanism of interaction between quasiparticles is independent of simple macroscopic parameters. For example, in some cases such notation as mass cannot be well defined at all (say, what is the mass of a phonon?).

The mathematical theory of transport processes is most advanced for mixtures of classical gases, the evolution of which is described by a set of Boltzmann equations. A basis of classical methods for solution of the Boltzmann equation in the case of a one-component gas is the formal expansion of distribution function in power series of some parameter σ in the form $f = f^{(0)} + \sigma f^{(1)} + \sigma^2 f^{(2)} + \cdots$, so that function $f^{(0)}$ corresponds to statistical equilibrium. In this case parameter σ is some scale factor for density, the physical sense of which can be different depending on a particular problem. As a rule, this parameter is formally considered to be small, so that the solution of the kinetic equation represents a problem of singular perturbation [7]. The most successful methods of solution of the kinetic equation, such as the Hilbert method and the Chapman-Enskog method [2,5], are based on this principle. In spite of the success of the Chapman-Enskog method in the description of connection between the kinetic theory and the equations of hydrodynamics (the Navier-Stokes equations appear already in the first order in parameter σ), the explicit expressions for kinetic coefficients have a rather complicated form. The main defect of these expressions is that already in the first order in parameter σ their analysis becomes practically impossible. The situation becomes more problematic in the case of a two-component gas. The infinite series of integral brackets containing Sonin polynomials does not allow one to select explicitly the contributions from different types of interactions in a system to various dissipative coefficients. This frequently leads to the necessity to use various ungrounded approximations, such as the Chapman-Cowling approximation [5] or the Kihara approximation [2].

At the same time, in spite of some successes [8], there is yet no consistent mathematical theory for deriving the dissipative coefficients in gases of quasiparticles. And, naturally, the problem of distinguishability of contribution from interactions between identical quasiparticles and between different subsystems has not been solved yet. This problem has its own history. In practice, when analyzing particular physical systems, many physicists use some model approximations for collision integrals. The most popular one is the so-called BGK approximation [9]. In its simple form the BGK approximation leads sometimes to quite confusing results. It

gives us several relaxation mechanisms in the system, and, therefore several characteristic times depending on momenta or energies of quasiparticles. The final observed quantities should be obtained by averaging this these in some manner. In particular problems the following question frequently appears: what must be averaged, i.e., the time or the rate (inverse time)? and how to obtain the real relaxation time, i.e., by summation of the times or by summation of the rates? This uncertainty led to many confusing situations. For example, for more than 15 years there were two different theories for the mobility of two-dimensional electron gas localized over the free surface of liquid helium [10]. The first theory [11] assumes the mobility to depend on the averaged characteristic time of electron-ripplon interaction (ripplons are the quantized surface waves of liquid helium). This theory well describes the experimental data for small electron density [12]. Another theory [13] assumes the mobility to be determined by the inverse averaged rate of the same interaction. These theoretical results fit well experiments [14] with large electron density. The problem of relationship between these two results has naturally appeared. There was a similar confusion in the theory of dissipative processes in superfluids [15]. The analogous situation took place for some time in the theory of thermal conductivity in solids [16].

The aim of this paper is to present an alternative approach for solution of the system of linearized kinetic equations for a two-component gas of quasiparticles with arbitrary statistics and dispersion law. The theory explicitly accounts for all types of interactions in the system. This allows one to analyze the contribution of interaction between identical particles to the relaxation of the whole system. I do not restrict myself to the frame of a particular system. So, the results obtained here can be applied to any quantum system, whose dissipative properties are determined by the processes in a two-component gas of quasiparticles.

The paper is organized as follows. In Sec. II I formulate the problem mathematically and carry out the linearization procedure. Section III is devoted to the procedure of inversion of the collision operator by the use of projection operator method. In Sec. IV I derive the exact solution for characteristic diffusion time and analyze all limiting cases. The classical systems of Lorentz and Rayleigh gases and their generalization to the quasiparticle systems are considered in Sec. V. In Sec. VI I consider the generalization of the Kihara approximation to the quasiparticle quantum systems. The outlines and conclusions are given in Sec. VII.

II. GENERAL EXPRESSIONS

Consider the stationary nonequilibrium state of a gaseous mixture of quasiparticles of two species. The most interesting relaxation process in such a system is a diffusion [17], so I will concentrate on the diffusive processes.

One of the most essential advantages of the offered theory is the fact that it is correct for quantum gases with any statistics and any dispersion of quasiparticles. All main outcomes remain valid for both systems with nonzero chemical potential, and with chemical potential equal to zero (when the number of quasiparticles is not conserved). The con-

structed theory does not meet principal difficulties in generalization to multicomponent systems (i.e., on systems with number of active components exceeding two).

The evolution of distribution functions f_k (subscript $k = 1,2$ numbers the components of the mixture) can be described by the following set of kinetic equations:

$$\mathbf{v}_k \frac{\partial f_k}{\partial \mathbf{r}} = \sum_{j=1,2} C_{kj}(f_k, f_j) \quad (k=1,2),$$
(1)

where $\mathbf{v}_k = \partial \epsilon_k / \partial \mathbf{p}_k$ is the velocity of a quasiparticle of the kth type, ϵ_k and $\mathbf{p}_k$ are its energy and momentum, respectively; $\mathbf{r}$ is the coordinate; and $C_{kj}(f_k, f_j)$ is the collision integral, which is a functional of the distribution functions of mixture components 1 and 2. The particular form of these collision integrals depends on the concrete physical problem. To find the diffusion coefficient let us consider the stationary nonequilibrium state of the two components 1 and 2 of the mixture, in which the quasiparticles' number densities are the functions of coordinate $\mathbf{r}$. In particular, for the gas of thermal excitations such situation can be realized by creation of a constant temperature gradient.

Under the considered conditions there are stationary gradients of partial pressure of components, which result in flow of quasiparticles. This flow is determined by momentum current density,

$$\mathbf{j}_k = -\sum_{j=1,2} \frac{\rho_k}{\rho} d_{kj} \frac{\partial P_j}{\partial \mathbf{r}} \quad (k=1,2),$$
(2)

where ρ_k is the normal density of the kth component of the system, $\rho = \rho_1 + \rho_2$ is the total density, and d_{kj} is the matrix of diffusion times. The partial pressure of quasiparticles is determined in the standard manner [5],

$$P_j = \frac{1}{3} \int \mathbf{p}_j \cdot \mathbf{v}_j f_j d\Gamma_j,$$
(3)

where $d\Gamma_j$ is the measure in phase space. The density of kth component can be written in the universal form [15],

$$\rho_k = -\frac{1}{3} \int \mathbf{p}_k^2 f_k' d\Gamma_k,$$
(4)

where $f_k' = \partial f_k / \partial \epsilon_k$. Relations (2)–(4) are suitable for the quasiparticles with arbitrary dispersion law, statistics, and chemical potential (I mean both for zero chemical potential and for nonzero one). Note that the definition of normal density (4) does not depend explicitly on such notations as mass or number density.

To exclude convective transport in a quasiparticle system, i.e., to investigate only dissipative processes, here after I shall consider the sum of partial pressures of different components of a mixture to be a constant, so put $P = P_1 + P_2 = $ const. The analytical relation between the matrix elements of a matrix of diffusion times d_{kj} and the usual diffusion coefficient D of a binary mixture can be obtained by a direct comparison of expression (2) with the definition of diffusion

coefficient. Thus, one can use the ordinary gas-dynamic definition of current density of the kth component of a mixture (see, e.g., Ref. [5]),

$$\mathbf{j}_k = \int \mathbf{p}_k f_k d\Gamma_k.$$
(5)

It appears that for various quantum and classical physical systems, the diffusion coefficient can be written in the most general form [15,18],

$$D = u_D^2 \tau_D,$$
(6)

where u_D is the characteristic velocity, whose analytic form depends on the particular dispersion law and statistics of quasiparticles, τ_D is the characteristic diffusion time to be determined.

According to the relations (2) and (5), for deriving the diffusion coefficient (6) it is necessary to solve the system of kinetic equations (1). Below, for definiteness, the diffusion in a system with conserved number of quasiparticles (and, therefore nonzero chemical potential) will be considered. The calculation for the case with nonconserved number of quasiparticles can be carried out within a similar framework.

Since we are interested in the theory within linear response approximation, let us assume the deviation of distribution functions f_k from their local equilibrium values $f_k^{(0)}$ to be small. So, put as usual,

$$f_k = f_k^{(0)} + \delta f_k, \quad |\delta f_k| \ll f_k^{(0)}.$$
(7)

The small deviation δf_k can be conveniently rewritten in the form

$$\delta f_k = -\frac{\partial f_k^{(0)}}{\partial \epsilon_k} g_k,$$
(8)

with unknown functions g_k. Linearizing the system of kinetic equations (1) we come to the system of linear integrodifferential equations for unknown quantities g_k, which determine the degree of system perturbation,

$$\frac{\mathbf{v}_k}{n_k} \frac{\partial P_k}{\partial \mathbf{r}} = C_{kk} g_k + C_{kj}(g_k + g_j) \quad (k,j=1,2; k \neq j).$$
(9)

Here n_k is the number density of the kth component, and C_{kj} are the linearized collision operators for the collisions within each component $(k=j)$ and for the collisions between different quasiparticles $(k \neq j)$. The acting of these operators on an arbitrary function of momentum, say $\xi(\mathbf{p}_{k,j})$, is determined by the particular form of collision integrals appeared in Eq. (1). If we deal with the ordinary binary collision integral with probability density function $W_{kj}(\mathbf{p}_k \cdot \mathbf{p}_j | \mathbf{p}_k' \cdot \mathbf{p}_j')$ then we obtain [19]

$$C_{kj}\xi(\mathbf{p}_{k,j}) = \int W_{kj}(\mathbf{p}_k \cdot \mathbf{p}_j | \mathbf{p}_k' \cdot \mathbf{p}_j') f_j^{(0)}(\mathbf{p}_j)$$

$$\times \{1 \pm f_k^{(0)}(\mathbf{p}_k)\}^{-1} \{1 \pm f_k^{(0)}(\mathbf{p}_k')\} \{1 \pm f_j^{(0)}(\mathbf{p}_j')\}$$

$$\times [\xi(\mathbf{p}_{k,j}') - \xi(\mathbf{p}_{k,j})] d\Gamma_j d\Gamma_k' d\Gamma_j' \quad (10)$$

for $k \neq j$, and

$$C_{kk}\xi(\mathbf{p}_k) = \int W_{kk}(\mathbf{p}_k \cdot \mathbf{p} | \mathbf{p}_k' \cdot \mathbf{p}') f_k^{(0)}(\mathbf{p}) \{1 \pm f_k^{(0)}(\mathbf{p}_k)\}^{-1}$$

$$\times \{1 \pm f_k^{(0)}(\mathbf{p}_k')\} \{1 \pm f_k^{(0)}(\mathbf{p}')\} [\xi(\mathbf{p}_k')$$

$$+ \xi(\mathbf{p}') - \xi(\mathbf{p}_k) - \xi(\mathbf{p})] d\Gamma d\Gamma_k' d\Gamma' \quad (11)$$

for $k = j$. The plus and minus signs in Eqs. (10) and (11) correspond to bosons and fermions, respectively.

Note that, in fact, our general approach allows us to account in such a manner not only for binary collisions, but for a variety of more specific types of interaction, such as decay or conversion processes [20], creation or annihilation of quasiparticles, interaction with boundaries and point defects, etc.

III. INVERSON OF THE COLLISION OPERATOR

According to relation (3), the gradient of partial pressure for quasiparticles with nonzero chemical potential can be written as

$$\frac{\partial P_k}{\partial \mathbf{r}} = n_k \left(\frac{\partial \mu_k}{\partial \mathbf{r}} \right)_{T = \text{const}}, \quad (12)$$

where μ_k is the chemical potential of the kth subsystem, and T is the mixture temperature.

For further calculations it is convenient to present system (9) in the compact matrix form,

$$\sum_{k=1,2} |\psi_k\rangle \frac{\partial P_k}{\partial \mathbf{r}} = \hat{C}|g\rangle, \quad (13)$$

where

$$|\psi_1\rangle = \left| \begin{matrix} \mathbf{v}_1 n_1^{-1} \\ 0 \end{matrix} \right\rangle, \quad |\psi_2\rangle = \left| \begin{matrix} 0 \\ \mathbf{v}_2 n_2^{-1} \end{matrix} \right\rangle, \quad |g\rangle = \left| \begin{matrix} g_1 \\ g_2 \end{matrix} \right\rangle, \quad (14)$$

are the two-component ket vectors, defined in an infinite Hilbert space to be specified. The collisional operator matrix $\hat{C}$ can be conveniently written as a sum

$$\hat{C} = \hat{J} + \hat{S}, \quad (15)$$

where the operator matrix

$$\hat{J} = \begin{pmatrix} C_{12} & C_{12} \\ C_{21} & C_{21} \end{pmatrix} \quad (16)$$

contains only the quantities corresponding to interactions between quasiparticles from different subsystems, and

$$\hat{S} = \begin{pmatrix} C_{11} & 0 \\ 0 & C_{22} \end{pmatrix} \quad (17)$$

corresponds to the relaxation inside each subsystem. To define completely the Hilbert space I am working in, let us introduce the scalar product of two-dimensional bra vector $\langle \zeta | = \langle \zeta_1(\mathbf{p}_1), \zeta_2(\mathbf{p}_2) |$ and ket vector $|\chi\rangle = (\langle \chi |)^\dagger$ in the following manner:

$$\langle \zeta | \chi \rangle = \sum_{k=1,2} (\zeta_k | \chi_k) = - \sum_{k=1,2} \int \zeta_k^* \chi_k \frac{\partial f_k^{(0)}}{\partial \epsilon_k} d\Gamma_k, \quad (18)$$

where $(\zeta_k |$ and $|\chi_k)$ are the corresponding one-component bra vector and ket vector, respectively. It is easy to verify that with such a choice of scalar product (18) the collision operator $\hat{C}$ becomes Hermitian.

System (13) is the system of nonuniform linear integral equations. According to the general theory of integral equations the sought solution $|g\rangle$ of system (13) must be orthogonal to the solution of corresponding uniform equations,

$$\hat{C}|\phi_1\rangle = 0. \quad (19)$$

The normalized solution of Eq. (19) can be written in the following form:

$$|\phi_1\rangle = \frac{1}{\sqrt{3\rho}} \left| \begin{matrix} \mathbf{p}_1 \\ \mathbf{p}_2 \end{matrix} \right\rangle. \quad (20)$$

This vector $|\phi_1\rangle$ corresponds to the total momentum of our two-component quasiparticle system. In general, Eq. (19) has other solutions, corresponding to conservation of energy, particle number, etc. I account here only for the momentum conservation law because the sought solution can depend only on the quasiparticle momenta [see initial equations (13)]. It is convenient to write the formal solution of Eq. (13) so that the orthogonality condition

$$\langle g | \phi_1 \rangle = 0 \quad (21)$$

is contained explicitly in the solution. For this purpose let us define the projection operator $\mathcal{P}_n$ onto the subspace orthogonal to the vector $|\phi_1\rangle$,

$$\mathcal{P}_n = 1 - \mathcal{P}_c, \quad \mathcal{P}_c = |\phi_1\rangle\langle \phi_1|. \quad (22)$$

As a result, the formal solution of Eq. (13) can be written in the form

$$|g\rangle = \mathcal{P}_n(\hat{C}^{-1})\mathcal{P}_n \sum_{k=1,2} |\psi_k\rangle \frac{\partial P_k}{\partial \mathbf{r}}. \quad (23)$$

Further, we must insert the solution (23) into the expression for current density (5), keeping in mind the relations (7) and (8). Comparing the obtained result with definition (2) for a matrix of diffusion times, we come to

$$d_{11} = \frac{\rho_2}{\rho_1} \tau_D, \quad d_{12} = d_{21} = \tau_D, \quad d_{22} = \frac{\rho_1}{\rho_2} \tau_D, \quad (24)$$

where

$$\tau_D = -\langle \phi_2 | \hat{\mathcal{C}}^{-1} | \phi_2 \rangle \tag{25}$$

is the characteristic diffusion time, and

$$|\phi_2\rangle = \frac{1}{\sqrt{3\rho\rho_1\rho_2}} \left| \begin{matrix} \rho_2 \mathbf{p}_1 \\ -\rho_1 \mathbf{p}_2 \end{matrix} \right\rangle \tag{26}$$

is the characteristic diffusion vector, which is orthogonal to $|\phi_1\rangle$.

Now the problem is reduced to calculation of the matrix element (25), which contains the inverse matrix operator determined by integral collision operators.

IV. EXACT AND LIMITING EXPRESSIONS FOR THE DIFFUSION TIME

To derive an exact, analytical expression for the unknown quantity (25) it is necessary to introduce a full system of orthonormal two-dimensional vectors $|\phi_n\rangle$ (here $n = 1,2,3,\ldots$) belonging to the infinite-dimensional Hilbert space with a scalar product (18). The concrete choice of a system of basis vectors in many respects depends on convenience of calculations within the framework of a concrete physical problem (see, e.g., Refs. [5] and [8]). In our problem it is convenient to take vector (20) as the first of them, and Eq. (26) as the second. The remaining vectors can be arbitrary (for example, such vectors can be built on the basis of Sonin polynomials in the classical case [5], or Akhiezer-Aleksin-Khodusov polynomials [8] in quantum systems), but should satisfy the completeness and orthogonality conditions,

$$\sum_{m=1}^{\infty} |\phi_m\rangle\langle\phi_m| = 1, \quad \langle\phi_m|\phi_n\rangle = \delta_{mn}. \tag{27}$$

In a constructed full system of vectors an exact expression for the diffusion time

$$\tau_D = -\{(\hat{J}+\hat{S})^{-1}\}_{22} \tag{28}$$

can be reduced to the visual analytical form allowing simple physical interpretation and providing a possibility to carry out the in-depth qualitative analysis of the obtained result in various limiting cases, corresponding to different mechanisms of equilibration in the system of quasiparticles,

$$\tau_D = -\left\{ I_{22} - \sum_{n,m=3}^{\infty} I_{2n}[(\mathcal{I}+\mathcal{S})^{-1}]_{nm} I_{m2} \right\}^{-1}. \tag{29}$$

Here the square matrices $\mathcal{I}$ and $\mathcal{S}$ contain the following matrix elements:

$$\|\mathcal{I}\|_{nm} = I_{nm} = \langle \phi_n | \hat{J} | \phi_m \rangle, \tag{30}$$

$$\|\mathcal{S}\|_{nm} = I_{nm} = \langle \phi_n | \hat{S} | \phi_m \rangle. \tag{31}$$

To obtain the formal result (29) I have used the relations (19) and (27).

The matrices (30) and (31) are infinite dimensional and nondiagonal. Therefore, the exact solution (29) does not allow one to obtain a closed analytical expression for the characteristic diffusion time. However, it is necessary to emphasize that the result (29) contains explicitly not only the quantities responsible for interaction between quasiparticles of different types, but also the matrix elements appropriate to collisions between the quasiparticles within each subsystem. It allows one to investigate various limiting cases, to find minimum and maximum values of diffusion time, to construct correct interpolation formulas and useful analytical models, and to make calculations on computers for concrete physical systems.

To carry out the detailed analysis of the formal solution (29), it is make explicit the vector basis $|\phi_n\rangle$ for $n>2$. Namely, let us choose the remaining vectors in the following form:

$$|\phi_{2\alpha+1}\rangle = \frac{1}{\mathcal{N}_1^{(\alpha)}} \left| \begin{matrix} F^{(\alpha)}(\mathbf{p}_1) \\ 0 \end{matrix} \right\rangle, \tag{32}$$

$$|\phi_{2\alpha+2}\rangle = \frac{1}{\mathcal{N}_2^{(\alpha)}} \left| \begin{matrix} 0 \\ F^{(\alpha)}(\mathbf{p}_2) \end{matrix} \right\rangle, \tag{33}$$

where $\alpha = 1,2,3,\ldots$, $F^{(\alpha)}(\mathbf{p}_k)$ is the system of properly chosen orthogonal polynomials with the norm $\mathcal{N}_k^{(\alpha)} = [F^{(\alpha)}(\mathbf{p}_k)|F^{(\alpha)}(\mathbf{p}_k)]^{1/2}$, such that $[F^{(\alpha)}(\mathbf{p}_k)|\mathbf{p_k}] = 0$ (it is clear that such polynomials cannot contain first degree of momentum). Such a choice (32), (33) provides a possibility to "separate" in some manner the components of the mixture. The matrix $\mathcal{J}+\mathcal{S}$ takes now the form

$$\mathcal{J}+\mathcal{S} = -\begin{pmatrix} \nu_1^{(11)} & \nu_{12}^{(11)} & \nu_1^{(12)} & \cdots \\ \nu_{21}^{(11)} & \nu_2^{(11)} & \nu_{21}^{(12)} & \cdots \\ \nu_1^{(21)} & \nu_{12}^{(21)} & \nu_1^{(22)} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$-\begin{pmatrix} \omega_1^{(11)} & 0 & \omega_1^{(12)} & \cdots \\ 0 & \omega_2^{(11)} & 0 & \cdots \\ \omega_1^{(21)} & 0 & \omega_1^{(22)} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \tag{34}$$

where I have introduced the following "higher" interaction rates:

$$\nu_{jk}^{(\alpha\beta)} = -\frac{[F^{(\alpha)}(\mathbf{p}_j)|C_{jk}|F^{(\beta)}(\mathbf{p}_k)]}{\mathcal{N}_j^{(\alpha)}\mathcal{N}_k^{(\beta)}} \tag{35}$$

and

$$\nu_j^{(\alpha\beta)} = -\frac{[F^{(\alpha)}(\mathbf{p}_j)|C_{jk}|F^{(\beta)}(\mathbf{p}_j)]}{\mathcal{N}_j^{(\alpha)}\mathcal{N}_j^{(\beta)}} \tag{36}$$

for collisions of different quasiparticles, and

$$\omega_j^{(\alpha\beta)} = -\frac{[F^{(\alpha)}(\mathbf{p}_j)|C_{jj}|F^{(\beta)}(\mathbf{p}_j)]}{\mathcal{N}_j^{(\alpha)}\mathcal{N}_j^{(\beta)}} \qquad (37)$$

corresponding to relaxation within each subsystem. The representation (34) helps us to understand the explicit structure of the formal solution (29). This provides a possibility to study the most important limiting cases and to reveal their physical significance.

So, in case of infinitely fast establishment of an equilibrium between quasiparticles of identical type (the so-called complete control regime [14,21]), when the strong inequalities

$$\omega_j^{(\alpha\beta)} \gg \nu_j^{(\alpha\beta)} \nu_{jk}^{(\alpha\beta)} \qquad (j,k=1,2) \qquad (38)$$

take place, the second term in the brackets of general result (29) vanishes and diffusion time (28) is given by the following simple formula [19]:

$$\tau_D^{(cc)} \equiv \tau_D^{(\min)} = -\frac{1}{I_{22}} = (\tau_{12}^{(0)-1} + \tau_{21}^{(0)-1})^{-1}, \qquad (39)$$

where

$$\tau_{kj}^{(0)-1} = -\langle C_{kj}\rangle_k^{-1} \qquad (k,j=1,2;k\neq j). \qquad (40)$$

Here and thereafter the brackets $\langle \cdots \rangle$ stand for a normalized average defined by the relation

$$\langle \mathcal{A}\rangle_k = \frac{1}{3\rho_k}(\mathbf{p}_k|\mathcal{A}|\mathbf{p}_k) \qquad (k=1,2). \qquad (41)$$

According to the momentum conservation law $C_{kj}|\mathbf{p}_k) = -C_{jk}|\mathbf{p}_j)$ the relation between "basic" interaction rates in Eq. (39) can be found as follows:

$$\tau_{12}^{(0)-1} = \frac{\rho_2}{\rho_1}\tau_{21}^{(0)-1}. \qquad (42)$$

Proceeding from Hermiticity and negativity of operators $\hat{S}$ and $\hat{J}$ with the help of the well-known Cauchy-Bunyakovsky inequality it is possible to show [19] that the following inequality is always valid:

$$\tau_D \geq \tau_D^{(\min)}, \qquad (43)$$

where $\tau_D^{(\min)}$ is determined by Eq. (39).

In the opposite limiting case of extremely slow establishment of equilibrium between identical quasiparticles, i.e., when

$$\omega_j^{(\alpha\beta)} \ll \nu_j^{(\alpha\beta)}, \nu_{jk}^{(\alpha\beta)} \qquad (j,k=1,2), \qquad (44)$$

the second matrix in Eq. (34) can be neglected and the whole formula (29) can be converted so that the diffusion time τ_D runs up to its maximum value,

$$\tau_D^{(\max)} = -\langle \phi_2|\hat{J}^{-1}|\phi_2\rangle. \qquad (45)$$

Comparing the main limiting results (39) and (45) we come to the following important conclusion: The main qualitative difference between expressions for diffusion time in case of a fast and slow relaxation inside each subsystem of a mixture consists in the method of averaging of operator matrices corresponding to interaction between *different* subsystems.

A very interesting situation can be realized when one, say first, subsystem equilibrates very slowly, $C_{11}\to 0$, but relaxation in another one is extremely, fast $C_{22}\to\infty$, so that

$$\omega_1^{(\alpha\beta)} \ll \nu_1^{(\alpha\beta)}, \nu_{12}^{(\alpha\beta)}, \qquad \omega_2^{(\alpha\beta)} \gg \nu_2^{(\alpha\beta)}, \nu_{21}^{(\alpha\beta)}. \qquad (46)$$

In this case the results (29) and (34) can be converted as follows:

$$\tau_D \approx -\langle C_{12}^{-1}\rangle_1 - \langle C_{21}\rangle_2^{-1}. \qquad (47)$$

Let us pay attention to the principal difference between the first and second terms in relation (47). Again, in the first term we average the time, while in the second one we average the rate of interaction between quasiparticles of different species. According to the Cauchy-Bunyakovsky inequality the characteristic time $\langle C_{jk}\rangle^{-1}$ is always less than the time $\langle C_{jk}^{-1}\rangle$ for any momentum dependence of C_{jk}. If the collision operator C_{jk} does not depend on the quasiparticles momenta, then $\tau_D \equiv \tau_D^{(\min)}$. If C_{jk} depends on momenta, then speed of equilibration of a system depends on how fast the equilibration between identical quasiparticles is [15,19]. Such a situation manifests itself in a phonon-impuriton system of superfluid mixtures of helium isotopes [22], in which the impuriton is the ^{3}He atom in superfluid ^{4}He, and in phonon systems in solids [16]. There is a two-stage mechanism of relaxation in these systems. At the first stage quasiparticles of the second type interact only with those quasiparticles of the first type, whose momenta correspond to maximum of collision operator C_{12}. At the second stage establishment of an equilibrium in a system is determined by the interaction of quasiparticles of first type with each other. This occurs because the quasiparticles of the first type with minimum C_{12} prefer to interact with such quasiparticles within their subsystem, which are already at equilibrium with the second subsystem.

The significance of the manner of averaging of collision operators can be illustrated in the simple example of lattice thermal conductivity in solids. The interaction rate between phonon and scattering center $\nu_{phi} \propto C_{phi}$ (say, a point defect or an impurity) is proportional to p_{ph}^4 (the so-called Rayleigh scattering). So, the integral $\langle C_{phi}^{-1}\rangle_{ph}$ diverges at zero momentum and the corresponding relaxation time tends to zero. This means that the nonequilibrium long wavelength phonon simply "does not see" an impurity. At the same time the quantity $\langle C_{phi}\rangle_{ph}$ is finite and leads to a finite thermal current. So, the mechanism of equilibration of such system can be drawn as follows: at first all phonons come to quasiequilibrium in their own subsystems, which corresponds to some stationary flux of phonons, then they begin to scatter on the impurities and point defects. As can be seen from the results

(29), (34), (39), (45), and (47), analogous competition mechanisms can occur in any two-component quasiparticle system.

Calculation of $\langle C_{jk} \rangle_j$ for particular physical systems does not meet any difficulties. However, to calculate $\langle C_{jk}^{-1} \rangle_j$ we must inverse the collision operator C_{jk}, which is not a straightforward operation. As a rule, it can be done by replacing C_{jk} with some characteristic rate of interaction between quasiparticles $\nu_{jk} = \nu_{jk}(\mathbf{p}_k)$, and then by straightforward averaging of the value $\nu_{jk}(\mathbf{p}_k)^{-1}$, which is simply a multiplying operator. In the followingt section I consider the most popular models in various problems associated with two-component classical gases or with condensed media whose transport properties are determined by the processes in two-component quasiparticle systems.

V. THE RAYLEIGH AND LORENTZ MODELS FOR TWO-COMPONENT GASEOUS MIXTURES

Any theory claiming for a solution of some complicated special problems, should first of all agree necessarily with some fundamental results in the most simple limiting cases. In the kinetic theory of classical gases the diffusion in a Lorentz gas (diffusion of a light, very rarefied component in a gas of massive, slow particles) and a Rayleigh gas (diffusion of massive particles with small concentration in a light gas) traditionally is considered.

Both classical models, i.e., the Rayleigh and Lorentz gases correspond, in fact, to the limiting case (46), but are more restricted in particle characteristics. Let us start with the Lorentz model. This is the mixture of a light component in very small concentration and a gas of massive, slow particles (buffer component). Let the light component be the first one. Let me briefly review the classical approach to a problem of diffusion in such a mixture [5,23]. The strong inequality

$$n_1/n_2 \ll 1 \qquad (48)$$

was required for light particles to interact only with a massive component, but not with one another. A large difference in the masses of particles,

$$m_1/m_2 \ll 1, \qquad (49)$$

ensured an elasticity of scattering of particles of a light component on massive particles and a large difference in thermal velocities of particles of different types. Thus, particles of the massive component can be treated as fixed and described by their equilibrium distribution function. With the purpose of calculating a diffusion coefficient in such a system the strong inequalities (48) and (49) are usually used for simplification of the initial kinetic equation, which can be reduced to the so-called Boltzmann-Lorentz equation [5,23]. The consequent solution of this equation gives an explicit expression for a diffusion coefficient.

Let us show that the classical results for diffusion in a Lorentz gas can be directly obtained from the general results given by Eqs. (29) and (34). The operations below have certain value themselves as generalizations of Rayleigh and Lorentz models on quantum gases of quasiparticles. Such a generalization is not trivial because of the impossibility to introduce the mass and conserved number density for some quasiparticles. So, we will formulate the problem in terms of normal densities. In this regard, for a Lorentz gas we replace two strong inequalities with the following:

$$\rho_1/\rho_2 \ll 1. \qquad (50)$$

Then we can rewrite the vector $|\phi_2\rangle$,

$$|\phi_2\rangle = \frac{1}{\sqrt{3\rho_1}} \left| \begin{matrix} \mathbf{p}_1 \\ 0 \end{matrix} \right\rangle \equiv \left| \begin{matrix} \chi_1 \\ 0 \end{matrix} \right\rangle. \qquad (51)$$

An equilibrium in the massive component leads to the following simple expression for operator matrix $\hat{J}$ given, in general, by Eq. (16):

$$\hat{J} = \begin{pmatrix} C_{12} & 0 \\ 0 & 0 \end{pmatrix}. \qquad (52)$$

Then, using the explicit expression (34) we can reduce the general result (29) to the following form:

$$\tau_D^{(L)} = \left\{ (\chi_1|C_{12}|\chi_1) - \frac{1}{\mathcal{N}_1^{(\alpha)}\mathcal{N}_1^{(\beta)}} \sum_{\alpha,\beta=1}^{\infty} [\chi_1|C_{12}|F^{(\alpha)}(\mathbf{p}_1)][\|(\phi_{2\alpha+1}|C_{12}|\phi_{2\beta+1})\|^{-1}]_{\alpha\beta}[F^{(\beta)}(\mathbf{p}_1)|C_{12}|\chi_1] \right\}^{-1}. \qquad (53)$$

The vectors $|\phi_{2\alpha+1})$ represent a complete set of orthonormal vector functions in the momentum space of the first component. This allows us to rewrite relation (53) as follows:

$$\tau_D^{(L)} = -\frac{1}{3\rho_1}(\mathbf{p}_1|C_{12}^{-1}|\mathbf{p}_1) \equiv \tau_{12}^{(\infty)}. \qquad (54)$$

Further simplification can be achieved if we believe the massive particles to be fixed during a collision, so that

$$v = |\mathbf{v}_1 - \mathbf{v}_2| \approx |\mathbf{v}_1|. \qquad (55)$$

This means that the differential cross section depends on the momenta of light particles only. Moreover, the momentum of a light particle can change only its direction but not an absolute value. With these assumptions we can write the collision operator C_{12} as

$$C_{12} = -|\mathbf{v}_1|\sigma_t n_2, \qquad (56)$$

which is simply a multiplication operator and, therefore, it can be inversed without any difficulty. Here I introduced the transport cross section [5]

$$\sigma_t = \int (1 - \cos\theta_{12})d\sigma, \qquad (57)$$

where θ_{12} is the scattering angle and $d\sigma$ is the ordinary differential cross section. As a result, using the relations (55)–(57) and defining the diffusion coefficient in a binary gas in the usual manner [5,23], we come to the classical result

$$D_{12}^{(L)} = \frac{1}{3nn_1} \int f_1^{(0)} \frac{v_1}{\sigma_t} d\Gamma_1. \qquad (58)$$

The obtained expression (58) coincides with analogous formulas obtained in Refs. [5] and [23] by simplification of the initial kinetic equation.

Note that for deriving the formula (58) in the frame of the approach developed here it is enough to require the fulfilment of strong inequality (50) for normal densities of components of a mixture, absence of equilibrium in the first component, equilibrium in the second component, and elasticity of quasiparticle scattering.

Now I shall consider one more classical example, namely, a diffusion in the so-called Rayleigh gas [24], that is, the diffusion of a very rarefied, massive gas in the light buffer component with large concentration. In other words, we, as in the previous case, have a mixture of light and massive components, but under opposite conditions. So we assume the concentration of a massive component to be so small that

$$\rho_2 / \rho_1 \ll 1. \qquad (59)$$

Now I assume that the light component of a mixture is already in equilibrium, while the particles of the massive component almost do not interact with one another. In this limiting case we can again use the result (53) with a replacement of subscripts $1 \leftrightarrow 2$. In this case, however, the relative velocity of particles from different components again is determined by the velocity of light particle. Therefore, the appropriate collision operator does not depend on momenta, so that in view of an orthogonality of vectors of selected basis its nongiagonal elements vanish, i.e.,

$$[\chi_2 | C_{21} | F^{(\beta)}(\mathbf{p}_2)] = 0 \quad (\beta = 1, 2, \dots). \qquad (60)$$

Thus, proceeding from the relation (53) with a replacement of subscripts of components, we come to the relation

$$\tau_D^{(R)-1} = -\frac{1}{3\rho_2}(\mathbf{p}_2 | C_{21} | \mathbf{p}_2) \equiv \tau_{21}^{(0)-1}, \qquad (61)$$

which determines the diffusion time in a weak solution of a massive component in an equilibrium light gas. Further, proceeding from a momentum conservation law, we obtain

$$\tau_D^{(R)-1} = -\frac{1}{3\rho_2}(\mathbf{p}_1 | C_{12}^{-1} | \mathbf{p}_1) = \frac{n_1 n_2}{3n\rho_2 T} \int f_1^{(0)} p_1^2 v_1 \sigma_t d\Gamma_1. \qquad (62)$$

Expression (62) leads to the well-known relation for the diffusion coefficient of massive particles in an equilibrium light classical gas [2,5]. Let me remark that in Ref. [5] this result was obtained by an indirect method with the use of the Einstein relation between the diffusion coefficient and mobility. The use of the method developed here has allowed to obtain the formula (62) without engaging artificial approaches, i.e., immediately from a general solution (29),(34). The developed approach allows to generalize the result (62), as well as Eq. (58), to quasiparticle systems with arbitrary dispersion law and statistics, in particular, to such systems, in which it is impossible to define notation of a mass in its classical sense.

VI. KIHARA APPROXIMATION

In the previous sections I considered only the most physically interesting limiting cases. However, in practice, the following problem can appear: how to calculate some dissipative coefficient more precisely in the intermediate case, i.e., when the considered limiting situations do not take occur. Of course, the most straightforward way is to compute it numerically using the formulas (29)–(34). In that case we are forced to restrict ourselves with some finite matrices in Eq. (29). This is an analog of the Chapman-Cowling approximation in classical gaseous mixtures. In 1949 Kihara [25] proposed another approximation, which is, in fact, simpler than Chapman-Cowling approximation. He simply proposed to neglect the nondiagonal integral brackets (see Ref. [2]), which is exact for Maxwell molecules. Unfortunately, this approximation is unproven until now and it can be partially justified only by experience in numerical calculations for classical gases. In our theory such an approximation can be introduced by neglecting all the nondiagonal matrix elements in Eq. (34). After this procedure the inverse matrix $(\mathcal{I} + \mathcal{S})^{-1}$ takes the form

$$(\mathcal{I} + \mathcal{S})^{-1} = -\begin{pmatrix} t_1^{(11)} & 0 & 0 & \dots \\ 0 & t_2^{(11)} & 0 & \dots \\ 0 & 0 & t_1^{(22)} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \qquad (63)$$

where I introduced the diagonal characteristic times

$$t_j^{(\alpha\beta)} = [\nu_j^{(\alpha\beta)} + \omega_j^{(\alpha\beta)}]^{-1}. \qquad (64)$$

In view of Eqs. (63) and (64) we can rewrite the result (29) in the following form:

$$\tau_D^{-1} \approx -\langle \phi_2 | \hat{\mathcal{C}} | \phi_2 \rangle + \frac{\langle \phi_2 | \hat{\mathcal{C}} | \phi_3 \rangle \langle \phi_3 | \hat{\mathcal{C}} | \phi_2 \rangle}{\langle \phi_3 | \hat{\mathcal{C}} | \phi_3 \rangle}$$

$$+ \frac{\langle \phi_2 | \hat{\mathcal{C}} | \phi_4 \rangle \langle \phi_4 | \hat{\mathcal{C}} | \phi_2 \rangle}{\langle \phi_4 | \hat{\mathcal{C}} | \phi_4 \rangle} + \cdots. \qquad (65)$$

The series expansion (65) gives the correction to the limiting result (39). In practice, we should estimate numerically an appropriate radius of convergence in series (65) and keep necessary number of terms.

In general, it is clear that the Kihara approximation works well when the momentum dependence of collision operators is weak enough. But sometimes series such as Eq. (65) have infinite radius of convergence. This indicates simply that it is better to start with the opposite limiting formula (45) as a zero order approximation.

VII. CONCLUSIONS

In the present work I present a general theory providing a possibility to investigate diffusion processes in a two-component gas of quasiparticles with arbitrary statistics and dispersion. The obtained main equations of the theory are correct for systems both with conserved and nonconserved number of quasiparticles, which is mathematically expressed in nonequality or in equality of a chemical potential with zero, accordingly. The proposed theory can be generalized to the cases of classical and quantum gaseous mixtures with arbitrary number of components.

To solve the formulated problem I start with the system of kinetic equations (1) driving the evolution of corresponding distribution functions of components of a mixture. After a standard procedure of a linearization (7),(9) of a kinetic problem, I have chosen the basis in infinite-dimensional two-parameter Hilbert space with a scalar product (18) selected so that the operator matrix of collision integrals (15)–(17) becomes Hermitian. As is known, the inverse matrix of collision integrals does not exist because of the moments of collision integrals. However, by projecting on the nucleus of an integral operator of collisions it is possible to define somewhat inverse matrix. In the present work this procedure has been made by introducing the projection operator (22) corresponding to conservation of total momentum of a quasiparticle system. As an outcome it allowed me to obtain the general expression (29) for characteristic time [see also Eq. (34)], determining speed of a diffusion relaxation in a system.

The obtained general result contains explicitly quantities responsible for interactions between quasiparticles from different components and between identical quasiparticles. It allows to analyze the qualitative difference between mechanisms of equilibration of a whole system in various limiting cases. So, if the relaxation inside each component of a system is instantaneous, the diffusion time is determined by an inverse average of collision operator, describing interaction between quasiparticles of different types. In the opposite limiting case, when the equilibrium in a system occurs over an infinite period of time, the diffusion time is equal to an average of the inverse collision operator. The principal difference between these two limiting results can be easily understood using an example of phonon thermal conductivity of solids. The thermal conductivity in this case is simply a diffusion of phonons in a system of fixed ''scatterers'' (impurities, boundaries, defects, etc.). If the phonons did not come yet to equilibrium with one another, the thermal conductivity is determined by average of the inverse frequency of scattering of phonons on scatterers. In case of a long wavelength phonon, such frequency is proportional to the fourth degree of momentum (Rayleigh scattering). Therefore, while averaging, the integral simply diverges at zero momentum. That is, the long wavelength phonon simply ''does not feel'' the scatterer. On the contrary, in the case when the phonons come in equilibrium with one another, the magnitude of the diffusion time appears to be finite, because now the frequency is averaged, instead of time. This corresponds to the so-called two-stage mechanism of relaxation. At first, an equilibrium in a phonon gas appropriate to some stationary phonon flux is established. Then phonons scatter on scatterers. Just this process results in a finite heat flux.

The general results, obtained in the work, give correct expressions for diffusion coefficients of a light gas in a massive one (Lorentz gas) (58) and a massive gas in a light one (Rayleigh gas) (62). Earlier, these classical results were obtained only with use of artificial methods based on simplification of the initial kinetic equation. Furthermore, these two most popular model systems can be easily generalized to describe quasiparticle systems as well as classical gases.

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